Exercise 35

For the following exercises, solve the equations over the complex numbers.

$$x^2 - 10x + 26 = 0$$

Solution

The two terms with x, x^2 and $10x$, cannot be combined, so it's necessary to complete the square to solve for x. Recall the following algebraic identity.

$$(x + B)^2 = x^2 + 2xB + B^2$$

Notice that $2B = -10$, which means $B = -5$ and $B^2 = 25$. Add and subtract 25 from the left side and apply the identity.

$$(x^2 - 10x + 25) + 26 - 25 = 0$$

$$(x + (-5))^2 + 1 = 0$$

$$(x - 5)^2 + 1 = 0$$

Now that x appears in only one place, it can be solved for. Subtract 1 from both sides.

$$(x - 5)^2 = -1$$

Take the square root of both sides.

$$\sqrt{(x-5)^2} = \sqrt{-1}$$

$$= i$$

Since there's an even power under an even root, and the result is to an odd power, an absolute value sign is needed around $x - 5$.

$$|x - 5| = i$$

Remove the absolute value sign by placing $\pm$ on the right side.

$$x - 5 = \pm i$$

Add 5 to both sides.

$$x = 5 \pm i$$

Therefore, $x = \{5 - i, 5 + i\}$.